ÉRETTSÉGI VIZSGA • 2023. május 9.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ
ÍRÁSBELI VIZSGA

minden vizsgázó számára

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **remark** in brackets, the solution should be considered complete without that remark as well.

6. Deduct points for **missing units** only if the missing unit is part of the answer or a unit exchange (without parentheses).
7. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. The score given for the solution of a problem, or part of a problem, **may never be negative**.
10. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
11. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
12. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
13. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
14. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
15. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1. a)

year	2013	2014	2015	2016	2017	2018	2019
ratio	2.92	2.20	2.18	1.34	1.33	2.13	2.00

(table above)	1 point	
The mean of the seven numbers is 2.01,	1 point	*Accept 2 as well.*
the standard deviation: $\sqrt{\frac{(2.92-2.01)^2+...+(2.00-2.01)^2}{7}}\approx$	1 point	*This point is also due if the candidate obtains the correct value of the standard deviation using a calculator.*
$\approx\sqrt{0.26}\approx 0.51$.	1 point	
Total:	**4 points**	

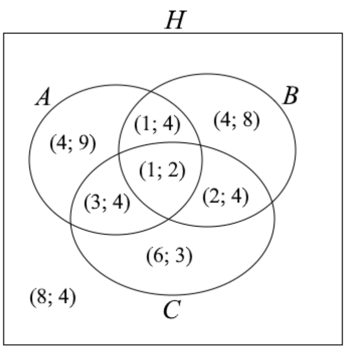
1. b)

The value given by the model: $c(6)=17.84\cdot 1.848^6\approx 711$ MW.	1 point	
$\frac{711}{640}\approx 1.11$,	1 point	
this is about 11% different from the actual value.	1 point	
Total:	**3 points**	

1. c)

$1.848^x=\frac{40\,000}{17.84}$ (≈ 2242.2)	1 point	
(The logarithm function is a one-to-one mapping, therefore) $x\cdot\log 1.848\approx\log 2242.2$ $x\approx\frac{\log 2242.2}{\log 1.848}$	2 points	$x=\log_{1.848}\frac{40\,000}{17.84}$
$x\approx 12.56$	1 point	
Total:	**4 points**	

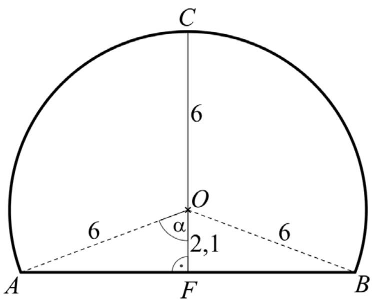
2. a)

An appropriate pair in each region, e.g. 	6 points	*Award 1 point for every appropriate pair (filling an empty region) also in the case of multiple pairs. Do not award points for regions that are filled with multiple solutions if there are incorrect ones among them.*
Total:	**6 points**	

2. b)		
Statement I is false.	1 point	
For example, 6 is a divisor of $2 \cdot 3$ and yet, 6 is a divisor of neither 2 nor 3.	1 point	
Statement II is false.	1 point	
For example, both 4 and 6 are divisors of 12 but the product $4 \cdot 6$ is not.	1 point	
Total:	**4 points**	

2. c)		
Converse: If c is a divisor of a or c is a divisor of b then c is a divisor of ab. Or: If c is not a divisor of ab then c is not a divisor of a **and** c is not a divisor of b.	1 point	
The converse is true.	1 point	
According to the premise, $a = kc$ or $b = mc$, and so $ab = (kb)c$ or $ab = (am)c$. Either way, ab is a multiple of c (c is a divisor of ab) ($k, m \in \mathbf{N}^+$).	2 points*	
Total:	**4 points**	

**Accept proofs that are less formal but otherwise correct.*

3. a)		
 Use the symbols of this diagram. (Point F is the midpoint of chord AB, $OA = OB = OC = 6$ m, $FC = 8.1$ m, $FO = FC - OC = 2.1$ m.)	1 point	
$\cos\alpha = \frac{OF}{OA} = 0.35$, and so $\alpha \approx 69.51°$.	2 points	
The central angle AOB: $360° - 2\alpha \approx 221°$.	1 point	
Total:	**4 points**	

3. b)		
The tunnel is considered to be a cylindrical object whose base is the vertical cross section and height is the $h = 340$ m length of the tunnel.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
(Using the symbols of the diagram in part a): the base area is the sum of the area t_1 of sector AOB with its 221° central angle, and the area t_2 of the triangle AOB.) $t_1 = \frac{221°}{360°} \cdot 6^2 \pi \approx 69.43 \text{ m}^2$,	2 points	
$2\alpha \approx 139°$, $t_2 = \frac{6^2 \cdot \sin 139°}{2} \approx 11.81 \text{ m}^2$.	2 points	$AF = \sqrt{AO^2 - OF^2} \approx$ ≈ 5.62 m, $t_2 = \frac{2AF \cdot OF}{2}$
The volume of the tunnel: $(t_1 + t_2) \cdot h \approx 27\,622 \text{ m}^3$,	1 point	
Rounded: 28 000 m^3.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**7 points**	

3. c)		
The surface tiled is the part of the lateral surface that belongs to the longer arc AB of the base circle. The length of this arc is $i = \frac{221°}{360°} \cdot 2 \cdot 6 \cdot \pi \approx 23.14$ m,	2 points	
and so the total area tiled is $i \cdot h = 23.14 \cdot 340 \approx 7868 \text{ m}^2$.	1 point	
Total:	**3 points**	

4. a) Solution 1		
The shopkeeper bought x eggs of size M last week, at f forints each, and he bought $(450 - x)$ size L eggs at $(f + 10)$ forints each. This week he bought $(450 - x)$ size M eggs, and x size L eggs: $\left.\begin{array}{r} f \cdot x + (f+10) \cdot (450 - x) = 25\,800 \\ f \cdot (450 - x) + (f+10) \cdot x = 23\,700 \end{array}\right\}$.	2 points	
Rearranged: $\left.\begin{array}{r} 450f + 4500 - 10x = 25\,800 \\ 450f + 10x = 23\,700 \end{array}\right\}$. Adding the equations: $900f = 45\,000$.	2 points	
$f = 50$, and substituting it gives $x = 120$.	1 point	

A size M egg costs 50 Ft, a size L egg costs 60 Ft, and last week the shopkeeper bought 120 size M eggs (and 330 size L eggs).	1 point	
Check: $50 \cdot 120 + 60 \cdot 330 = 25\,800$ and $50 \cdot 330 + 60 \cdot 120 = 23\,700$.	1 point	
Total:	**7 points**	

4. a) Solution 2		
If the shopkeeper had bought the same number of size L and size M eggs, he would have paid the same both weeks. As he paid more on week one, he must have bought more large eggs then.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The difference between the number of large and medium eggs bought last week times 10 Ft is the extra he paid last week, compared to this week.	1 point	
$(25\,800 - 23\,700) : 10 = 210$, so there were this many more large eggs bought last week.	1 point	
So he must have bought $(450 - 210) : 2 = 120$ medium eggs last week (and 330 large ones).	1 point	
If a medium egg costs f forints then $120f + 330(f + 10) = 25\,800$,	1 point	
so the cost of a medium egg is $f = 50$ Ft, the cost of a large egg is 60 Ft.	1 point	
Check: $50 \cdot 120 + 60 \cdot 330 = 25\,800$ and $50 \cdot 330 + 60 \cdot 120 = 23\,700$.	1 point	
Total:	**7 points**	

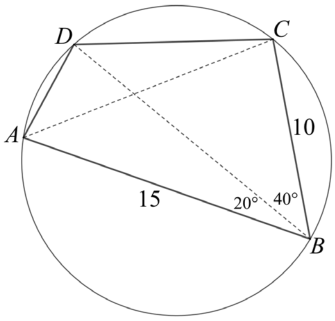
4. a) Solution 3		
Let f be the price of a medium egg, then a large one costs $f + 10$ Ft. The shopkeeper bought 450 of both kinds over these two weeks, so $450f + 450(f + 10) = 25\,800 + 23\,700$.	2 points	
$900f = 45\,000$,	1 point	
$f = 50$ Ft for each medium egg, and $50 + 10 = 60$ Ft for each large egg.	1 point	
Assuming the shopkeeper bought x medium eggs last week and $(450 - x)$ large ones then $50x + 60(450 - x) = 25\,800$,	1 point	
$x = 120$, this is the number of medium eggs bought last week (while the number of large eggs is 330).	1 point	
Check: $50 \cdot 120 + 60 \cdot 330 = 25\,800$ and $50 \cdot 330 + 60 \cdot 120 = 23\,700$.	1 point	
Total:	**7 points**	

4. b) **Solution 1**		
Balázs can have his scrambled eggs if either the first or the second egg is bad (because then he still has enough good ones to work with) or if the first 4 eggs are all good.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The probability that the first egg is bad is $\frac{1}{6}$.	1 point	
The probability that the first egg is good but the second is bad is $\frac{5}{6}\cdot\frac{1}{5}=\frac{1}{6}$.	1 point	
The probability that the first four eggs are all good is $\frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}=\frac{2}{6}$.	1 point	
The probability that Balázs can have his scrambled eggs is $\frac{1}{6}+\frac{1}{6}+\frac{2}{6}=\frac{2}{3}$.	1 point	
Total:	**5 points**	

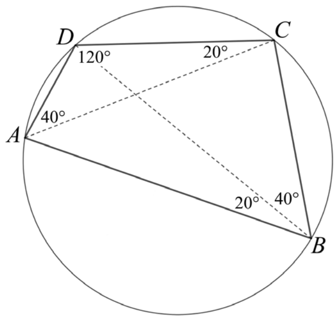
4. b) **Solution 2**		
Balázs can have his scrambled eggs if either the first or the second egg is bad (because then he still has enough good ones to work with) or if the first 4 eggs are all good.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
There are a total 6! (=720) different orders to break the six eggs.	1 point	
We are looking for those where the bad egg is first, second, fifth or sixth. There are 5! different arrangement of each of these cases.	1 point	
The number of favourable cases is then $4\cdot5!$ (= 480).	1 point	
The probability is $\frac{4\cdot5!}{6!}=\frac{4}{6}\left(=\frac{2}{3}\right)$.	1 point	
Total:	**5 points**	

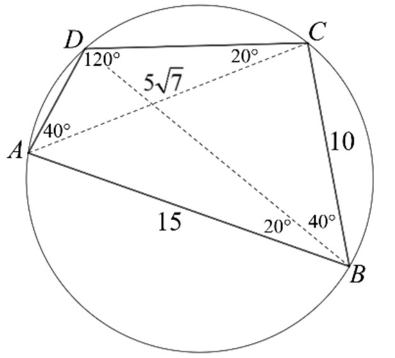
4. b) **Solution 3**		
Balázs can have his scrambled eggs if either the first or the second egg is bad (because then he still has enough good ones to work with) or if the first 4 eggs are all good.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The bad egg can show up on any of the six positions at the same probability of $\frac{1}{6}$ (as all positions are equally likely). The cases where the bad egg shows up on position 1, 2, 5 or 6 are favourable.	3 points	*Unfavourable are the cases where the bad egg shows up on positions 3 or 4. The probability of this is* $2\cdot\frac{1}{6}=\frac{1}{3}$.
The probability is $\frac{4}{6}\left(=\frac{2}{3}\right)$.	1 point	$1-\frac{1}{3}=\frac{2}{3}$
Total:	**5 points**	

II.

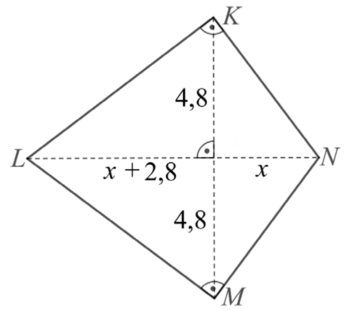
5. a)		
 (The angle at vertex B in triangle ABC is 60°.) Law of Cosines in triangle ABC: $AC^2 = 15^2 + 10^2 - 2\cdot 15\cdot 10\cdot \cos 60°$.	1 point	
$AC^2 = 225 + 100 - 150 = 175$.	1 point	
$AC = \sqrt{175} = \sqrt{25\cdot 7} = 5\cdot\sqrt{7}$.	1 point	
Total:	**3 points**	

Note: Award a maximum of 2 points if the candidate uses an approximation while calculating AC.

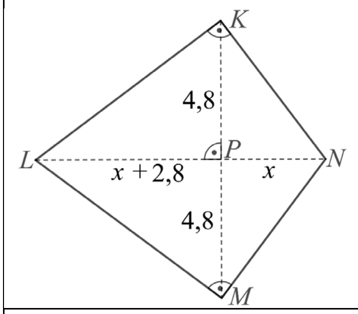
5. b)		
The quadrilateral $ABCD$ is cyclic, so $ADC\sphericalangle = 180° - 60° = 120°$.	1 point	
As stated by the theorem about central and inscribed angles: $CAD\sphericalangle = 180° - 120° - 20° = 40°$. (The statement is true.)	1 point	
Total:	**2 points**	

5. c)		
 Law of Sines in triangle ACD: $\frac{AD}{AC}=\frac{\sin 20°}{\sin 120°}$.	1 point	$\frac{CD}{AC}=\frac{\sin 40°}{\sin 120°}$
$AD=5\sqrt{7}\cdot\frac{\sin 20°}{\sin 120°}\approx 5.22$	1 point	$CD\approx 9.82$
The area of quadrilateral $ABCD$ is the sum of the areas of triangles ABC and ACD: $\frac{15\cdot 10\cdot \sin 60°}{2}+\frac{5\sqrt{7}\cdot 5.22\cdot \sin 40°}{2}\approx$ $\approx (64.95+22.19\approx)\ 87.1.$	2 points	
Total:	**4 points**	

Note: the semiperimeter of the cyclic quadrilateral is s ≈ 20.02, its area is (use the formula for the area of cyclic quadrilaterals) $\sqrt{5.02\cdot 10.02\cdot 10.20\cdot 14.80}\approx 87.1$.

5. d) Solution 1		
 The sections of the diagonal that is the line of symmetry are x cm and $x + 2.8$ cm long.	1 point	*Kite KLMN is cyclic (see Thales' theorem).*
Use the height theorem in the right triangle KLN: $x(x+2.8)=4.8^2$.	2 points	*The product of lengths of the sections of any chord through the point of intersection of the diagonals is constant:* $x(x+2.8)=4.8^2$.
Rearrange: $x^2+2.8x-23.04=0$.	1 point	
The real roots of the equation are 3.6 and –6.4, the latter is obviously wrong here.	1 point	
The diagonal that is the line of symmetry is $3.6+6.4=10$ cm long,	1 point	
the area is $(9.6\cdot 10:2=)\ 48\ \text{cm}^2$.	1 point	
Total:	**7 points**	

5. d) Solution 2

 Pythagoras' theorem in the right triangle *NKL*: $NK^2 + KL^2 = (2x + 2.8)^2$, in the right triangle *NPK*: $NK^2 = x^2 + 4.8^2$, in the right triangle *KPL*: $KL^2 = (x + 2.8)^2 + 4.8^2$.	2 points	*The right triangles LPK and KPN are similar, because their corresponding acute angles are equal (their sides are perpendicular in pairs).*
$x^2 + 4.8^2 + (x + 2.8)^2 + 4.8^2 = (2x + 2.8)^2$.	1 point	*The equality of the corresponding ratios gives* $\frac{NP}{KP} = \frac{KP}{LP}$, *i.e.* $\frac{x}{4.8} = \frac{4.8}{x + 2.8}$.
$2x^2 + 5.6x + 53.92 = 4x^2 + 11.2x + 7.84$ $2x^2 + 5.6x - 46.08 = 0$	1 point	$x^2 + 2.8x - 23.04 = 0$
The real roots of the equation are 3.6 and –6.4, the latter is obviously wrong here.	1 point	
(E.g. Pythagoras' theorem) *KN* = 6 cm, *KL* = 8 cm,	1 point	
the area of the kite is $\frac{KN \cdot KL}{2} \cdot 2 = 48$ cm^2.	1 point	
Total:	**7 points**	

6. a) **Solution 1**		
Number the seats e.g. from left to right. The following, non-adjacent, possibilities are available for the three girls: 1-3-5, 1-3-6, 1-3-7, 1-4-6, 1-4-7, 1-5-7, 2-4-6, 2-4-7, 2-5-7, 3-5-7.	2 points*	*Award 1 point if the candidate makes 1 mistake, 0 points for 2 or more mistakes.*
In each of these the girls may sit in 3! (= 6) different ways.	1 point	
The 4 boys may fill the rest of the seats in 4! (= 24) different ways.	1 point	
The total number of arrangement is then 10 · 3! · 4! =	1 point	
= 1440.	1 point	
Total:	**6 points**	

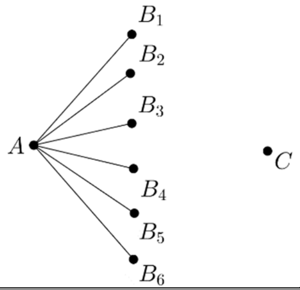
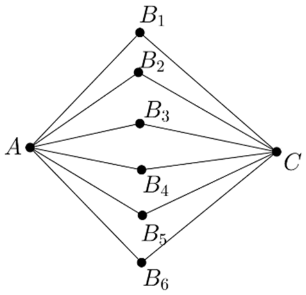
*Note: Award the points marked * for the following reasoning, too: If neither the girls nor the boys are distinguished, then the three girls determines four ‘gaps’, marked with x-s (*x *G* x *G* x *G* x*). At least one boy must sit in each of the two middle gaps, to separate the girls. The remaining two boys must select one each from the four gaps, such that the order of selection is not important and any gap may be selected multiple times. This is the number of combinations of two items out of four with repeat:* $\binom{4+2-1}{2}=\binom{5}{2}=10$.

6. a) **Solution 2**		
(Determine the number of arrangements where two girls do not sit in adjacent seats.) There are 4! (= 24) different orders for the four boys to sit.	1 point	
In any arrangement of the boys the positions for the 3 girls may be selected from among the 5 options marked x in the line-up: x B x B x B x B x.	1 point	
This gives $\binom{5}{3}$ (= 10) different possibilities.	1 point	*The first girls may choose from among 5 options, the second has 4, the third 3. This makes 5·4·3 (=60) different options.*
In each case the girls may sit in 3! (= 6) different orders.	1 point	
The total number of arrangements is: $4!\cdot\binom{5}{3}\cdot 3!=$	1 point	$4!\cdot 5\cdot 4\cdot 3$
= 1440.	1 point	
Total:	**6 points**	

6. b) **Solution 1**		
Mark the members of the party, in order of ascending heights, *A, B, C, D, E* and *F*. The shortest is *A*, who must sit in one of the 3 seats in the first row.	1 point	
Any of the remaining 5 people may sit behind *A*. This is $3 \cdot 5$ (= 15) cases.	1 point	
The shortest of the remaining four people must, again, sit in one of the 2 remaining seats in the first row.	1 point	
Any of the remaining 3 people may sit behind, which gives $2 \cdot 3$ (= 6) cases.	1 point	
There is only one way for the remaining 2 people to sit,	1 point	
so the final number of cases is $15 \cdot 6 \cdot 1 = 90$.	1 point	
Total:	**6 points**	

6. b) **Solution 2**		
((Number the seats behind one another: 1-2, 3-4, 5-6.) Pairs of people are seated behind one another. The first two people (on seats 1-2) may be selected in $\binom{6}{2}$(= 15) different ways. These two may only sit down one way, the taller must sit behind the shorter one.	2 points	
The next two people may be selected in $\binom{4}{2}$(= 6) different ways and may only sit down one way (on seats 3-4).	2 points	
The last two people may sit down on the last two seats (5-6) in only one way.	1 point	
The final number of cases is $15 \cdot 6 \cdot 1 = 90$.	1 point	
Total:	**6 points**	

6. b) **Solution 3**		
There are a total 6! (=720) ways for the six people to sit down at all.	2 points	
The first two people may sit behind one another in two different ways but only one of these (half of the cases) will be correct. The same applies to the second and third pair, too, only half of the possible seating arrangements will work.	2 points	
The final number of correct arrangements is therefore $\frac{6!}{2 \cdot 2 \cdot 2} = 90$.	2 points	
Total:	**6 points**	

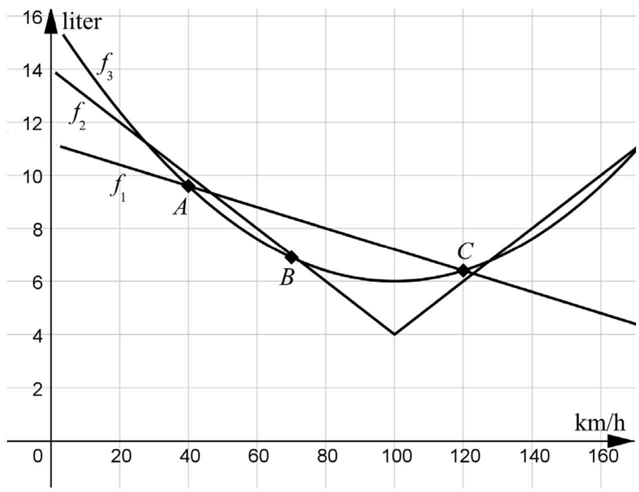
6. c)		
 Let vertex A be of degree 6 and let the vertices connected to A be B_1, B_2, … , B_6. Let C be the 8th vertex of the graph. Draw the 6 edges from A.	1 point	
 (As CA is not an edge) the remaining 7 edges must all be in the subgraph $\{C, B_1, B_2, \dots, B_6\}$. As (seen in the diagram) there may be only 6 edges drawn from C, so there must be at least one edge in the subgraph $\{B_1, B_2, \dots, B_6\}$, e.g. B_1B_2.	2 points	*If there is an edge in the subgraph $\{B_1, B_2, \dots, B_6\}$, e.g. B_1B_2, then AB_1B_2 is a triangle, so the statement is proven. If there is no edge in this subgraph then all the remaining 7 edges would have to start from C which is impossible (as this is a simple graph and CA is not an edge).*
In this case AB_1B_2 forms a triangle, proving the original statement.	1 point	
Total:	**4 points**	

7. a)		
The car has travelled 20 kilometres at 40 km/h and another $120 \cdot \frac{5}{6} = 100$ kilometres at 120 km/h. This is a total 120 kilometres.	2 points	
On these 120 kilometres the car consumed $0.2 \cdot 9.6 + 6.4 = 8.32$ litres of fuel.	1 point	
The average fuel consumption on 100 km was $\frac{8.32}{1.2} \approx 6.93$ litres for this leg of the route.	1 point	
Total:	**4 points**	

7. b)		
$f_1(40) = 9.6; f_1(70) = 8.4; f_1(120) = 6.4,$	1 point	
so $\lvert f_1(40) - 9.6\rvert + \lvert f_1(70) - 6.9\rvert + \lvert f_1(120) - 6.4\rvert =$ $(= 0 + 1.5 + 0) = 1.5.$	1 point	
$f_2(40) = 10; f_2(70) = 7; f_2(120) = 6,$	1 point	
so $\lvert f_2(40) - 9.6\rvert + \lvert f_2(70) - 6.9\rvert + \lvert f_2(120) - 6.4\rvert =$ $(= 0.4 + 0.1 + 0.4) = 0.9.$	1 point	
The function f_2 is a better estimate.	1 point	
Total:	**5 points**	

7. c)		
($f_3(40) = 9.6$ so) $1600a + 40b + c = 9.6$. (1) ($f_3(70) = 6.9$, so) $4900a + 70b + c = 6.9$. (2) ($f_3(120) = 6.4$, so) $14\,400a + 120b + c = 6.4$. (3)	2 points	
(Solve the system of three variables and three equations.) Subtract (2) from (1): $-3300a - 30b = 2.7$ Subtract (2) from (3): $9500a + 50b = -0.5$	1 point	*Subtract* (1) *from* (3)*:* $12\,800a + 80b = -3.2$. *Subtract* (1) *from* (2)*:* $3300a + 30b = -2.7$.
Multiply the first equation by 30 and the second by 50: $-110a - b = 0.09$ $190a + b = -0.01$	1 point	*Express b from both equations:* $b = -0.04 - 160a$, *and* $b = -0.09 - 110a$.
Add these: $80a = 0.08$, $a = 0.001$.	1 point	$-0,04 - 160a =$ $= -0.09 - 110a$ $50a = 0,05$ $a = 0.001$
Use substitution to find $b = -0.2$, and $c = 16$. (So $f_3(x) = 0.001x^2 - 0.2x + 16$, which fits perfectly on all three data points.)	2 points	
Total:	**7 points**	

Note: The three data points (A, B, C) and the best estimate functions (f_1, f_2, f_3) are shown in the diagram below:



8. a)		
The probability of 100 on Wheel 1 is $\frac{2}{5} = 0.4$ (the probability of any other outcome is 0.6).	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The probability of exactly four 100-s is $\binom{10}{4} \cdot 0.4^4 \cdot 0.6^6$,	1 point	
which is approximately 0.251.	1 point	
Total:	**3 points**	

8. b)		
The probability of winning 200 forints is $\frac{2}{5} \cdot \frac{1}{4} = 0.1$ (both wheels stopping at 100), the probability of winning 400 forints is $\frac{2}{5} \cdot \frac{2}{4} = 0.2$ (both wheels stopping at 200), the probability of winning 1600 forints is $\frac{1}{5} \cdot \frac{1}{4} = 0.05$ (both wheels stopping at 800).	2 points	
The expected value of what the player **wins** is $0.1 \cdot 200 + 0.2 \cdot 400 + 0.05 \cdot 1600 = 180$ Ft.	2 points*	
The average expected **gain** is $180 – 200 = –20$ Ft.	1 point*	
Total:	**5 points**	

*Note: The points marked * may also be given for the following reasoning:*

The probability of the player not winning is $1 – (0.1 + 0.2 + 0.05) = 0.65$.	1 point	
The player may **gain** (–200) Ft, 0 Ft, 200 Ft or 1400 Ft at the respective probabilities of 0.65, 0.1, 0.2, 0.05.	1 point	
The average expected gain is then: $0.65 \cdot (–200) + 0.1 \cdot 0 + 0.2 \cdot 200 + 0.05 \cdot 1400 = –20$ Ft.	1 point	

8. c)		
To achieve a “bingo” one wheel must stop at 200, the other at 800. The probability of a “bingo” is then $\frac{2}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{2}{4} =$	2 points	
$= 0.2$ indeed.	1 point	
Total:	**3 points**	

8. d)		
The probability of not achieving a "bingo" is 0.8.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
Assume a player turns the wheels n times. The probability of none of them being a "bingo" is 0.8^n. The probability of at least one "bingo" is $1 - 0.8^n$.	1 point	
Here $1 - 0.8^n \geq 0.95$, that is $0.8^n \leq 0.05$.	1 point*	
As the base 0.8 exponential function is strictly monotone decreasing, $n \geq \log_{0.8} 0.05$.	1 point*	$n \cdot \log 0.8 \leq \log 0.05$ *Dividing by the negative number* $\log 0.8$*:* $n \geq \frac{\log 0.05}{\log 0.8}$.
$n \geq 13.4$, so a minimum 14 games are required.	1 point*	
Total:	**5 points**	

*Note: Award 1 out of the 3 points marked * if the candidate correctly solves the equation* $1 - 0.8^n = 0.95$. *Award a further 1 point if they use their reasoning to give the correct answer (minimum 14 games).*

9. a)		
Function f has a local extreme where $f'(x) = 0$,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
i.e. at $x = 2$ or $x = 5$.	1 point	
At $x = 2$ the derivative function keeps its sign, so it is not a local extreme.	1 point	
At $x = 5$ the derivative function changes from negative to positive,	1 point	
so it is a local minimum.	1 point	
Total:	**5 points**	

9. b)		
$f'(x) = x^3 - 9x^2 + 24x - 20$	1 point	
$f(x) \in \int f'(x)\ dx$	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$f(x) = \frac{x^4}{4} - 3x^3 + 12x^2 - 20x + c$	1 point	
(As the graph of the function passes through the point (0; 1) so) $f(0) = 1$,	1 point	
therefore $c = 1$ (which means $f(x) = \frac{x^4}{4} - 3x^3 + 12x^2 - 20x + 1$).	1 point	
Total:	**5 points**	

9. c) **Solution 1**		
It must be proven that $g'(x) > 0$ is true for all $x \in \mathbf{R}$. Then the monotone increase of g will follow.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
Use the quotient-rule: $g'(x) = \dfrac{(3x^3+x)'(x^2+1)-(3x^3+x)(x^2+1)'}{(x^2+1)^2} =$	1 point	
$= \dfrac{(9x^2+1)(x^2+1)-(3x^3+x)\cdot 2x}{(x^2+1)^2} =$	1 point	
$= \dfrac{(9x^4+10x^2+1)-(6x^4+2x^2)}{(x^2+1)^2} =$ $= \dfrac{3x^4+8x^2+1}{(x^2+1)^2}$.	1 point	
The numerator is positive (1 or more) and the denominator is the square of a positive number.	1 point	
This makes the whole fraction positive, therefore $g'(x) > 0$ is true.	1 point	
Total:	**6 points**	

9. c) **Solution 2**		
Let $a < b$ ($a, b \in \mathbf{R}$). $g(b) - g(a) = \dfrac{3b^3+b}{b^2+1} - \dfrac{3a^3+a}{a^2+1} =$ $= \dfrac{(3b^3+b)(a^2+1)-(3a^3+a)(b^2+1)}{(a^2+1)(b^2+1)}$.	1 point	
The numerator, after expanding the parentheses, combining the like terms and factoring: $3(b^3-a^3)+3a^2b^2(b-a)-ab(b-a)+b-a =$	1 point	
$= (b-a)(3b^2+3ab+3a^2+3a^2b^2-ab+1) =$	1 point	
$= (b-a)\left[2b^2+2a^2+(a+b)^2+3a^2b^2+1\right]$.	1 point	
Both factors of the product, and therefore the whole numerator, is positive. The denominator is also positive (the product of two positive numbers),	1 point	
therefore *g(b) – g(a)* is always positive, so the statement is true.	1 point	
Total:	**6 points**	